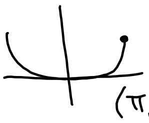
9.3 Taylor series with remainder

What is the fifth order Maclaurin series for f(x)=sin(x)? What is the maximum error when approximating sin(x) on $[-\pi,\pi]$

Solve graphically and numerically

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} + \text{~~Remainder~~ error}$$

$$\epsilon = \left|\sin x - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\right)\right|$$



$(\pi, .524)$

Lagrange error $M=1$

$$\epsilon \le \left|\frac{M \cdot x^7}{7!}\right|$$

Max error $= .524 < \left|\frac{\pi^7}{7!}\right| = .599$

Jan 31-5:30 PM

How many terms are needed in the Maclaurin series for sin(x) in order to approximate sin(x) within .0001 on the interval $[-\pi,\pi]$

$a=0$

$M=1$

$x=\pi$

$$\left|\frac{M x^{n+1}}{(n+1)!}\right| < .0001$$

$$\frac{\pi^{n+1}}{(n+1)!} < .0001$$

$n=\cancel{13}$ $\epsilon=.000105$

$n=14$ $\epsilon=.000022$

Jan 31-6:00 PM

On what interval does the third order Maclaurin series approximate sin(x) within .012

find x

~~n=3~~

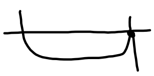
n=4

ϵ

$$x - \frac{x^3}{3!} + 0 \cdot \frac{x^4}{4!}$$

$[-1.037, 1.037]$

$$\left|\frac{x^5}{5!}\right| < .01$$



1.037

$$\frac{x^5}{5!} < .01$$

$$x < \sqrt[5]{.01(5!)} = 1.037$$

Jan 31-6:02 PM

Jan 20-10:13 AM

Taylor's Remainder Estimation Theorem

(Lagrange)

interval order

$$\epsilon \le \left| \frac{M (x-a)^{n+1}}{(n+1)!} \right| \qquad M = \text{Max of } f^{(n+1)}(x)$$

If Taylor:

$$f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} \cdots \frac{f^{(n)}(a)(x-a)^n}{n!}$$

Jan 31-6:03 PM

The approximation $\ln(1+x) \approx x - \frac{x^2}{2}$ is used when x is small.

Use the Remainder Estimation Theorem to get a bound for the maximum error when $|x| \le .01$ Support the answer graphically.

Jan 31-6:07 PM